Singular Integrals And Differentiability Properties Of Functions

About fifty years ago S. G. Mikhlin, in solving the regularization problem for two-dimensional singular integral operators [56], assigned to each such operator a function which he called a symbol, and showed that regularization is possible if the infimum of the modulus of the symbol is positive. Later, the notion of a symbol was extended to multidimensional singular integral operators (of arbitrary dimension) [57, 58, 21, 22]. Subsequently, the synthesis of singular integral, and differential operators [2, 8, 9] led to the theory of pseudodifferential operators [17, 35] (see also [35(1)-35(17)])*, which are naturally characterized by their symbols. An important role in the construction of symbols for many classes of operators was played by Gelfand's theory of maximal ideals of Banach algebras [201. Using this theory, criteria were obtained for Fredholmness of one-dimensional singular integral operators with continuous coefficients [34 (42)], Wiener-Hopf operators [37], and multidimensional singular integral operators [38 (2)]. The investigation of systems of equations involving such operators has led to the notion of matrix symbol [59, 12 (14), 39, 41]. This notion plays an essential role not only for systems, but also for singular integral operators with piecewise-continuous (scalar) coefficients [44 (4)]. At the same time, attempts to introduce a (scalar or matrix) symbol for other algebras have failed. This textbook provides a self-contained and elementary introduction to the modern theory of pseudodifferential operators and their applications to partial differential equations. It presents the necessary material on Fourier transformation and distribution theory, the basic calculus of pseudodifferential operators on the n-dimensional Euclidean space, an introduction to the theory of singular integral operators, the modern theory of Besov and Bessel potential spaces, and several applications to wellposedness and regularity question for elliptic and parabolic equations. The basic notation of functional analysis needed in the book is introduced and summarized in the appendix.

The origins of the harmonic analysis go back to an ingenious idea of Fourier that any reasonable function can be represented as an infinite linear combination of sines and cosines. Today's harmonic analysis incorporates the elements of geometric measure theory, number theory, probability, and has countless applications from data analysis to image recognition and from the study of sound and vibrations to the cutting edge of contemporary physics. The present volume is based on lectures presented at the summer school on Harmonic Analysis. These notes give fresh, concise, and high-level introductions to recent developments in the field, often with new arguments not found elsewhere. The volume will be of use both to graduate students seeking to enter the field and to senior researchers wishing to keep up with current developments.

This book develops a new theory of multi-parameter singular integrals associated with Carnot-Carathéodory balls. Brian Street first details the classical theory of Calderón-Zygmund singular integrals and applications to linear partial differential equations. He then outlines the theory of multi-parameter Carnot-Carathéodory geometry, where the main tool is a quantitative version of the classical theorem of Frobenius. Street then gives several examples of multi-parameter singular integrals arising naturally in various problems. The final chapter of the book develops a general theory of singular integrals that generalizes and unifies these examples. This is one of the first general theories of multi-parameter singular integrals that goes beyond the product theory of singular integrals and their analogs. Multi-parameter Singular Integrals will interest graduate students and researchers working in singular integrals and related fields.

This book is a collection of papers reflecting the conference held in Caracas, Venezuela, in January 1994 in celebration of Professor Mischa Cotlar's eightieth birthday. Presenting an excellent account of recent advances in harmonic analysis and operator theory and their applications, many of the contributors are world leaders in their fields. The collection covers a broad spectrum of topics, including: wavelet analysis, Haenkel operators, multimeasure theory, the boundary behavior of the Bergman kernel, interpolation theory, and Cotlar's Lemma on almost orthogonality in the context of L[superscript p] spaces and more... The range of topics in this volume promotes cross-pollination among the various fields covered. Such variety makes "Harmonic Analysis and Operator Theory" an inspiration for graduate students interested in this area of study.

'A Century In Books' chronicles the 100-year history of the Princeton University Press and highlights 100 of the nearly 8000 books it has produced over the past century. This proceedings volume presents 36 papers given by leading experts during the Third Conference on Function Spaces held at Southern Illinois University at Edwardsville. A wide range of topics in the subject area are covered. Most papers are written for nonexperts, so the book can serve as a good introduction to the topic for those interested in this area. The book presents the following broad range of topics, including spaces and algebras of analytic functions of one and of many variables, $L^p$ spaces, spaces of Banach-valued functions, isometries of function spaces, geometry of Banach spaces and related subjects. Known results, open problems, and new discoveries are featured. At the time of publication, information about the book, the conference, and a list and pictures of contributors are available on the Web.

The main purpose of this book is to provide a detailed and comprehensive survey of the theory of singular integrals and Fourier multipliers on Lipschitz curves and surfaces, an area that has been developed since the 1980s. The subject of singular integrals and the related Fourier multipliers on Lipschitz curves and surfaces has an extensive background in harmonic analysis and partial differential equations. The book elaborates on the basic framework, the Fourier methodology, and the main results in various contexts, especially addressing the following topics: singular integral operators with holomorphic kernels, fractional integral and differential operators with holomorphic kernels, holomorphic and monogenic Fourier multipliers, and Cauchy-Dunford functional calculi of the Dirac operators on Lipschitz curves and surfaces, and the high-dimensional Fueter mapping theorem with applications. The book offers a valuable resource for all graduate students and researchers interested in singular integrals and Fourier multipliers.

This book introduces some important progress in the theory of CalderonOCoZygmund singular integrals, oscillatory singular integrals, and LittlewoodOCoPaley theory over the last decade. It includes some important research results by the authors and their cooperators, such as singular integrals with rough kernels on Block spaces and Hardy spaces, the criterion on boundedness of oscillatory singular integrals, and boundedness of the rough Marcinkiewicz integrals. These results have frequently been cited in many published papers."

This volume contains a selection of papers on modern operator theory and its applications, arising from a joint workshop on linear one-dimensional singular integral equations. The book is of interest to a wide audience in the mathematical and engineering sciences.
The book discusses the extensions of basic Fourier Analysis techniques to the Clifford algebra framework. Topics covered: construction of Clifford-valued wavelets, Calderon-Zygmund theory for Clifford valued singular integral operators on Lipschitz hyper-surfaces, Hardy spaces of Clifford monogenic functions on Lipschitz domains. Results are applied to potential theory and elliptic boundary value problems on non-smooth domains. The book is self-contained to a large extent and well-suited for graduate students and researchers in the areas of wavelet theory, Harmonic and Clifford Analysis. It will also interest the specialists concerned with the applications of the Clifford algebra machinery to Mathematical Physics.

Systematically constructing an optimal theory, this monograph develops and explores several approaches to Hardy spaces in the setting of Ahlfors-regular quasi-metric spaces. The text is divided into two main parts, with the first part providing atomic, molecular, and grand maximal function characterizations of Hardy spaces and formulates sharp versions of basic analytical tools for quasi-metric spaces, such as a Lebesgue differentiation theorem with minimal demands on the underlying measure, a maximally smooth approximation to the identity and a Calderon-Zygmund decomposition for distributions. These results are of independent interest. The second part establishes very general criteria guaranteeing that a linear operator acts continuously from a Hardy space into a topological vector space, emphasizing the role of the action of the operator on atoms. Applications include the solvability of the Dirichlet problem for elliptic systems in the upper-half space with boundary data from Hardy spaces. The tools established in the first part are then used to develop a sharp theory of Besov and Triebel-Lizorkin spaces in Ahlfors-regular quasi-metric spaces. The monograph is largely self-contained and is intended for mathematicians, graduate students and professionals with a mathematical background who are interested in the interplay between analysis and geometry.

This book contains an expanded version of lectures delivered by the authors at the CRM in Spring of 2009. It contains four series of lectures. The first one is an application of harmonic analysis and the Heisenberg group to understand human vision. The second and third series of lectures cover some of the main topics on linear and multilinear harmonic analysis. The last one is a clear introduction to a deep result of De Giorgi, Moser and Nash on regularity of elliptic partial differential equations in divergence form. Authored by a ranking authority in Gaussian harmonic analysis, this book embodies a state-of-the-art entrée at the intersection of two important fields of research: harmonic analysis and probability. The book is intended for a very diverse audience, from graduate students all the way to researchers working in a broad spectrum of areas in analysis. Written with the graduate student in mind, it is assumed that the reader has familiarity with the basics of real analysis as well as with classical harmonic analysis, including Calderón-Zygmund theory; also some knowledge of basic orthogonal polynomials theory would be convenient. The monograph develops the main topics of classical harmonic analysis (semigroups, covering lemmas, maximal functions, Littlewood-Paley functions, spectral multipliers, fractional integrals and fractional derivatives, singular integrals) with respect to the Gaussian measure. The text provide an updated exposition, as self-contained as possible, of all the topics in Gaussian harmonic analysis that up to now are mostly scattered in research papers and sections of books; also an exhaustive bibliography for further reading. Each chapter ends with a section of notes and further results where connections between Gaussian harmonic analysis and other connected fields, points of view and alternative techniques are given. Mathematicians and researchers in several areas will find the breadth and depth of the subject highly useful.

Under minimal assumptions on a function $\psi$ we obtain wavelet-type frames of the form $\psi_{j,k}(x) = r^{\frac{j}{2}}(1/2^n) \psi(r^j x - sk), j \in \mathbb{N}, k \in \mathbb{N}^n, s$ for some $r > 1$ and $s > 0$. This collection is shown to be a frame for a scale of Triebel-Lizorkin spaces (which includes Lebesgue, Sobolev and Hardy spaces) and the reproducing formula converges in norm as well as pointwise a.e. The construction follows from a characterization of those operators which are bounded on a space of smooth molecules. This characterization also allows us to decompose a broad range of singular integral operators in terms of smooth molecules.

The second part of a two-volume set concerning the field of Clifford (geometric) algebra, this work consists of thematically organized chapters that provide a broad overview of cutting-edge topics in mathematical physics and the physical applications of Clifford algebras: from applications such as complex-distance potential theory, supersymmetry, and fluid dynamics to Fourier analysis, the study of boundary value problems, and applications, to mathematical physics and Schwarzian derivatives in Euclidean space. Among the mathematical topics examined are generalized Dirac operators, holonomy groups, monogenic and hypermonogenic functions and their derivatives, quaternionic Beltrami equations, Fourier theory under Mobius transformations, Cauchy-Reimann operators, and Cauchy type integrals.

Singular Integrals and Differentiability Properties of Functions (PMS-30)Princeton University Press

Quadrature domains were singled out about 30 years ago by D. Aharonov and H.S. Shapiro in connection with an extremal problem in function theory. Since then, a series of coincidental discoveries put this class of planar domains at the center of crossroads of several quite independent mathematical theories, e.g., potential theory, Riemann surfaces, inverse problems, holomorphic partial differential equations, fluid mechanics, operator theory. The volume is devoted to recent advances in the theory of quadrature domains, illustrating well the multi-facet aspects of their nature. The book contains a large collection of open problems pertaining to the general theme of quadrature domains.

Based on a graduate course given by the author at Yale University this book deals with complex analysis (analytic capacity), geometric measure theory (rectifiable and uniformly rectifiable sets) and harmonic analysis (boundedness of singular integral operators on Ahlfors-regular sets). In particular, these notes contain a description of Peter Jones' geometric traveling salesman theorem, the proof of the equivalence between uniform rectifiability and boundedness of the Cauchy operator on Ahlfors-regular sets, the complete proofs of the Denjoy conjecture and the Vitushkin conjecture (for the latter, only the Ahlfors-regular case) and a discussion of X. Tolsa's solution of the Painlevé problem.

Revised edition of a first-year graduate course on probability theory. This book is a comprehensive introduction to the mathematical theory of vorticity and incompressible flow ranging from elementary introductory material to current research topics. While the contents center on mathematical theory, many parts of the book showcase the interaction between rigorous mathematical theory, numerical, asymptotic, and qualitative simplified modeling, and physical phenomena. The first half forms an introductory graduate course on vorticity and incompressible flow. The second half comprises a modern applied mathematics graduate course on the weak solution theory for incompressible flow.

In this book we suggest a unified method of constructing near-minimizers for certain important functionals arising in approximation, harmonic analysis and ill-posed problems and most widely used in interpolation theory. The constructions
are based on far-reaching refinements of the classical Calderón–Zygmund decomposition. These new Calderón–Zygmund decompositions in turn are produced with the help of new covering theorems that combine many remarkable features of classical results established by Besicovitch, Whitney and Wiener. In many cases the minimizers constructed in the book are stable (i.e., remain near-minimizers) under the action of Calderón–Zygmund singular integral operators. The book is divided into two parts. While the new method is presented in great detail in the second part, the first is mainly devoted to the prerequisites needed for a self-contained presentation of the main topic. There we discuss the classical covering results mentioned above, various spectacular applications of the classical Calderón–Zygmund decompositions, and the relationship of all this to real interpolation. It also serves as a quick introduction to such important topics as spaces of smooth functions or singular integrals.

Singular integrals are among the most interesting and important objects of study in analysis, one of the three main branches of mathematics. They deal with real and complex numbers and their functions. In this book, Princeton professor Elias Stein, a leading mathematical innovator as well as a gifted expositor, produced what has been called the most influential mathematics text in the last thirty-five years. One reason for its success as a text is its almost legendary presentation: Stein takes arcane material, previously understood only by specialists, and makes it accessible even to beginning graduate students. Readers have reflected that when you read this book, not only do you see that the greats of the past have done exciting work, but you also feel inspired that you can master the subject and contribute to it yourself. Singular integrals were known to only a few specialists when Stein's book was first published. Over time, however, the book has inspired a whole generation of researchers to apply its methods to a broad range of problems in many disciplines, including engineering, biology, and finance. Stein has received numerous awards for his research, including the Wolf Prize of Israel, the Steele Prize, and the National Medal of Science. He has published eight books with Princeton, including Real Analysis in 2005.

This volume is dedicated to the eminent Russian mathematician I.B. Simonenko on the occasion of his 70th birthday. It presents recent results in Fredholm theory for singular integral and convolution operators, estimates for singular integral operators on Carleson curves acting in $L^p$ spaces with variable exponents, the finite sections method for band-dominated and Toeplitz operators, Szegö type theorems, the averaging method for nonlinear equations, among others. This volume contains translations of papers that originally appeared in the Japanese journal Sugaku. The papers range over a variety of topics, including differential equations with free boundary, singular integral operators, operator algebras, and relations between the Brownian motion on a manifold with function theory. The volume is suitable for graduate students and research mathematicians interested in analysis and differential equations.

This book explores several important aspects of recent developments in the interdisciplinary applications of mathematical analysis (MA), and highlights how MA is now being employed in many areas of scientific research. Each of the 23 carefully reviewed chapters was written by experienced expert(s) in respective field, and will enrich readers' understanding of the respective research problems, providing them with sufficient background to understand the theories, methods and applications discussed. The book's main goal is to highlight the latest trends and advances, equipping interested readers to pursue further research of their own. Given its scope, the book will especially benefit graduate and PhD students, researchers in the applied sciences, educators, and engineers with an interest in recent developments in the interdisciplinary applications of mathematical analysis.

This book addresses key aspects of recent developments in applied mathematical analysis and its use. It also highlights a broad range of applications from science, engineering, technology and social perspectives. Each chapter investigates selected research problems and presents a balanced mix of theory, methods and applications for the chosen topics. Special emphasis is placed on presenting basic developments in applied mathematical analysis, and on highlighting the latest advances in this research area. The book is presented in a self-contained manner as far as possible, and includes sufficient references to allow the interested reader to pursue further research in this still-developing field. The primary audience for this book includes graduate students, researchers and educators; however, it will also be useful for general readers with an interest in recent developments in applied mathematical analysis and applications.

Discrete decomposition techniques for spaces for functions or distributions are very useful tools for studying many problems in analysis. In this work, the author uses this type of decomposition to analyze a large class of operators, including Calderon-Zygmund operators.

This volume gives an account of the current state of weight theory for integral operators, such as maximal functions, Riesz potential, singular integrals and their generalization in Lorentz and Orlicz spaces. Starting with the crucial concept of a space of homogeneous type, it continues with general criteria for the boundedness of the integral operators considered, then address special settings and applications to classical operators in Euclidean spaces.

This book illustrates the wide range of research subjects developed by the Italian research group in harmonic analysis, originally started by Alessandro Figà-Talamanca, to whom it is dedicated in the occasion of his retirement. In particular, it outlines some of the impressive ramifications of the mathematical developments that began when Figà-Talamanca brought the study of harmonic analysis to Italy; the research group that he nurtured has now expanded to cover many areas. Therefore the book is addressed not only to experts in harmonic analysis, summability of Fourier series and singular integrals, but also in potential theory, symmetric spaces, analysis and partial differential equations on Riemannian manifolds, analysis on graphs, trees, buildings and discrete groups, Lie groups and Lie algebras, and even in far-reaching applications as for instance cellular automata and signal processing (low-discrepancy sampling, Gaussian noise).

This volume, dedicated to Bernd Silbermann on his sixtieth birthday, collects research articles on Toeplitz matrices and singular integral equations written by leading area experts. The subjects of the contributions include Banach algebraic methods, Toeplitz determinants and random matrix theory, Fredholm theory and numerical analysis for singular integral equations, and efficient algorithms for linear systems with structured matrices, and reflect Bernd Silbermann's broad spectrum of research interests. The volume also contains a biographical essay and a list of publications. The book is addressed to a wide audience in the mathematical and engineering sciences. The articles are carefully written and are accessible to motivated readers with basic knowledge in functional analysis and operator theory.
The Third International Workshop on Complex Structures and Vector Fields was held to exchange information on current topics in complex analysis, differential geometry and mathematical physics, and to find new subjects in these fields. This volume contains many interesting and important articles in complex analysis (including quaternionic analysis), functional analysis, topology, differential geometry (hermitian geometry, surface theory), and mathematical physics (quantum mechanics, hamilton mechanics).

In just over 100 pages, this book provides basic, essential knowledge of some of the tools of real analysis: the Hardy–Littlewood maximal operator, the Calderón–Zygmund theory, the Littlewood–Paley theory, interpolation of spaces and operators, and the basics of H1 and BMO spaces. This concise text offers brief proofs and exercises of various difficulties designed to challenge and engage students. An Introduction to Singular Integrals is meant to give first-year graduate students in Fourier analysis and partial differential equations an introduction to harmonic analysis. While some background material is included in the appendices, readers should have a basic knowledge of functional analysis, some acquaintance with measure and integration theory, and familiarity with the Fourier transform in Euclidean spaces.

The present edition differs from the original German one mainly in the following additional material: weighted norm inequalities for maximal functions and singular operators (§12, Chap. XI), polysingular integral operators and pseudo-differential operators (§§ 7, 8, Chap. XII), and spline approximation methods for solving singular integral equations (§4, Chap. XVII). Furthermore, we added two subsections on polynomial approximation methods for singular integral equations over an interval or with discontinuous coefficients (Nos. 3.6 and 3.7, Chap. XVII). In many places we incorporated new results which, in the vast majority, are from the last five years after publishing the German edition (note that the references are enlarged by about 150 new titles). S. G. Mikhlin wrote §§ 7, 8, Chap. XII, and the other additions were drawn up by S. Prossdorf. We wish to express our deepest gratitude to Dr. A. Bottcher and Dr. R. Lehmann who together translated the text into English carefully and with remarkable expertise.

This volume contains the proceedings of the International Workshop on Operator Theory and Applications held at the University of Algarve in Faro, Portugal, September 12-15, in the year 2000. The main topics of the conference were: Operator Theoretical Methods in Diffraction Theory; Algebraic Techniques in Operator Theory; Applications to Mathematical Physics and Related Topics. A total of 94 colleagues from 21 countries participated in the conference. The major part of participants came from Portugal (32), Germany (17), Israel (6), Mexico (6), the Netherlands (5), USA (4) and Austria (4). The others were from Ukraine, Venezuela (3 each), Spain, Sweden (2 each), Algeria, Australia, Belgium, France, Georgia, Italy, Japan, Kuwait, Russia and Turkey (one of each country). It was the 12th meeting in the framework of the IWOTA conferences which started in 1981 on an initiative of Professors I. Gohberg (Tel Aviv) and J. W. Helton (San Diego). Up to now, it was the largest conference in the field of Operator Theory in Portugal.